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When investigating waves created by the explosion of a charge of explosive in solid media, we use real, instantaneous wave and instantaneous waveless detonation schemes.

In the case of initiation at the center of a spherical charge of explosives, a spherical detonation wave is formed. At the instant of arrival of its front at the surface of the charge, a spherical shock wave (jump) arises in the surrounding medium, while along the products of detonation toward the center there begins to move a converging wave which can be a rarefication wave or a shock wave, dependent on the characteristics of the explosive matter and the medium. As a result of addition of the elementary waves moving from the boundary of the gas chamber, at the center a diverging wave is created. In the process of repeated motions of the wave widening of the gas chamber, evening out, and reduction in the pressure of the products of detonation take place. The scheme of real detonation takes into account these wave processes.

Simpler is the scheme of instantaneous wave detonation. It is assumed that the entire charge detonates instantaneously. In the surrounding medium there arises a shock wave, while toward the center of the charge from the contact discontinuity along the region of constant pressure there moves a rarefication wave. At the center, just as in the scheme of real detonation, elementary waves interact and there arises a new spherical diverging wave. The process of wave changes is repeated, the pressure in the gas chamber is evened out and reduced, the front of the shock wave moves away from the boundary of the gas chamber.

According to the scheme of instantaneous waveless detonation, the charge detonates instantaneously, the pressure in the entire volume of the explosion cavity is the same (is evened out instantaneously) and varies only with time [1-5].

In the schemes of real and instantaneous wave detonation under consideration, we assume a single isentropic equation of state of the products of explosion. The following single-term equation [6] is widely used:

$$
\begin{equation*}
p=p_{n}\left(\rho / \rho_{n}\right)^{k} \tag{1}
\end{equation*}
$$

where $\mathrm{P}_{\mathrm{n}}$ and $\rho_{\mathrm{n}}$ are the pressure and density at the front of the detonation wave.
More accurately, the properties of the products of detonation are described by the twoterm equation $[1,2,7]$

$$
\begin{equation*}
p=A \rho^{n}+B \rho^{\rho+1} \tag{2}
\end{equation*}
$$

It is assumed that for high pressure this equation is transformed into (1), while for low pressure it has the form

$$
\begin{equation*}
p=p_{0}\left(\rho / \rho_{0}\right)^{k_{0}}, \tag{3}
\end{equation*}
$$

where $p_{0}$ is the atmospheric pressure, $\rho_{0}$ is the corresponding density of the products of explosion.

At the front of the detonation wave the internal energy of the products of detonation $E$ is equal to the sum of the heat of explosion $Q$ and the energy of shock transition

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$$
E=Q+\left(p_{n}-p_{0}\right)\left(V_{n}-V_{0}\right) / 2 .
$$

The quantities $A, B, n$ and $\gamma$ are determined from the following condition; for the pressure corresponding to the front of the detonation wave, the curves (1) and (2) have a common tangent and a point, for $\rho \rightarrow 0$ the curves (1) and (3) have a common tangent, when the pressure extends from $P_{n}$ to $p o$ the products of detonation carry out work equal to $E$.

From these conditions together with (2) we obtain the system for determining $A, B$, $\mathrm{n}, \boldsymbol{\gamma}$

$$
\begin{gathered}
k=n+B \rho_{n}^{\gamma+1}(\gamma+1-n) / p_{n}, \quad \gamma=k_{0}-1, \\
Q+\left(p_{n}-p_{0}\right)\left(V_{n}-V_{0}\right) / 2=\frac{p_{n}}{\rho_{n}(n-1)}+\frac{B \rho_{n}^{\gamma}(n-1-\gamma)}{\gamma(n-1)} .
\end{gathered}
$$

We use the Lagrange variables $r$ (a spatial coordinate in units of length) and $t$ (time).
In terms of these variables, the basic equations of motion of solid medium have the form

$$
\begin{equation*}
\frac{\partial V}{\partial t}-\frac{1}{\rho_{0}}\left(\frac{R}{r}\right)^{v} \frac{\partial u}{\partial r}-\frac{v u V}{R}=0, \quad \frac{\partial u}{\partial t}-\frac{1}{\rho_{0}}\left(\frac{R}{r}\right)^{v} \frac{\partial p}{\partial r}=0 \tag{4}
\end{equation*}
$$

where $R$ is the Euler coordinate and $\nu=2$.
System (4) is closed by Eq. (2) for the region of the products of detonation and by an equation determining the behavior of the surrounding medium (water or water-saturated ground) for the region of the shock wave. The solution is determined in the medium and in the products of detonation.

The boundary conditions are: At the front of the shock wave in the medium

$$
p-p_{0}=\rho_{0} u D,\left(\rho-\rho_{0}\right) D=\rho u
$$

on the contact explosion (the boundary of the gas chamber) the pressure and velocity of particles in the products of detonation and in the medium are equal, at the center of symmetry the velocity of particles is zero.

Calculations have been carried out for pentolyte and trotyl. Here it was taken: For trotyl $D=6950 \mathrm{~m} / \mathrm{sec}, \rho_{0}=1600 \mathrm{~kg} / \mathrm{m}^{3}, \mathrm{Q}=10^{6} \mathrm{cal} / \mathrm{kg}, \mathrm{k}=3$, $\mathrm{k}_{0}=1.25$, for pentolyte $\mathrm{D}=7650 \mathrm{~m} / \mathrm{sec}, \rho_{\rho}=1650 \mathrm{~kg} / \mathrm{m}^{3}, \mathrm{Q}=1.218 \cdot 10^{6} \mathrm{cal} / \mathrm{kg}, \mathrm{k}=2.94, \mathrm{k}_{0}=1.26$ ( D is the velocity of the detonation wave $\left.\mathrm{P}_{\mathrm{n}}=\rho_{0} \mathrm{D} /(\mathrm{k}+1)\right)$.

We consider waves in water and water-saturated ground as barotropic media (entropy does not enter in explicit form). To water the compressibility equation of Tate is applicable. To water-saturated ground the equation of a three-component medium without viscosity [2] is app1icable:

$$
\begin{gathered}
\frac{\rho_{0}}{\rho}=\sum_{i=1}^{3} \alpha_{i}\left[\frac{\gamma_{i}\left(p-p_{0}\right)}{\rho_{i 0} c_{i 0}^{2}}+1\right]^{-\frac{1}{\gamma_{i}}} \\
\rho_{0}=\sum_{i=1}^{3} \alpha_{i} \rho_{i 0}, \quad \alpha_{1}+\alpha_{2}+\alpha_{3}=1
\end{gathered}
$$

where the quantities with the index 1 refer to air trapped in the pores of the ground, 2 refers to water in pores, and 3 to the hard component (quartz); $\alpha_{i}$, content of the i-th component by volume; $\rho_{i o}$, density; $c_{i o}$, velocity of sound in the i-th component with $p=p_{0}$.

The calculations have been presented for $\alpha_{1}=0.04, \alpha_{2}=0.36, \alpha_{3}=0.60$, which correspond to a water-saturated ground with an average content of trapped air [3]. We have assumed $\rho_{10}=1.29 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{20}=1000 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{30}=2650 \mathrm{~kg} / \mathrm{m}^{3}, c_{10}=330 \mathrm{~m} / \mathrm{sec}, c_{20}=1480$ $\mathrm{m} / \mathrm{sec}, c_{30}=4500 \mathrm{~m} / \mathrm{sec}, \gamma_{1}=1.4, \gamma_{2}=7.15, \gamma_{3}=4$. In the Tate equations the same values of the quantities $\gamma_{2}, c_{20}$, and $\rho_{20}$ have been taken.

The solution is carried out by the characteristics method used earlier in [1, 2, 8]. In the problem under consideration in the $R$, $t$ plane there are several types of points; at


Fig. 1
each of them the calculation is conducted according to its own algorithm: at the front of the shock wave $S$ in the medium, between $S$ and the contact explosion $T$ (the boundary of the gas chamber), on $T$, in the products of detonation, and at the center of symmetry. The step along the spatial coordinate is taken in accordance with the Courant criterion.

According to the two schemes thus adopted we have calculated the parameters of the wave in water in the case of an explosion of a charge of pentolyte, in order to compare them with the results of [9], where the solution has been carried out by the finite-differences method with artificial viscosity according to the scheme of a real detonation and a nonbarotropic equation of state of the products of explosion. For water the nonbarotropic equation

$$
p=f_{1} / V+f_{2} / V^{3}+f_{3} / V^{j}+f_{4} / V^{3}
$$

is used, where $f_{2}, f_{2}, f_{3}$ and $f_{4}$ are polynomial functions of internal energy.
In accordance with the scheme of a real detonation, at the instant when the front of the detonation wave reaches the boundary surface, the distribution of parameters behind the front is characterized by two zones: the zone of rest, from the center of symmetry to $\overline{\mathrm{R}}=$ $0.458 \overline{\mathrm{R}}_{0}$, where the velocity of particles is zero, while the pressure is $56.9 \cdot 10^{\mathrm{B}} \mathrm{N} / \mathrm{m}^{2}$, and the nonstationary portion from the zone of rest to the front of the detonation wave. The velocity of particles at the front is $1942 \mathrm{~m} / \mathrm{sec}$, while the pressure is $250.15 \cdot 10^{8} \mathrm{~N} / \mathrm{m}^{2}$. According to the scheme of instantaneous wave detonation, the pressure over the entire explosion cavity is $125 \cdot 10^{8} \mathrm{~N} / \mathrm{m}^{2}$, while the velocity of particles is zero [10].

In Fig. 1 we have shown graphs of the maximum pressure of the wave depending on the dimensionless distance $\bar{R}=R / R$ ) ( $R_{0}$ is the radius of the $\operatorname{explosive}$ charge) in water. Here and in what is to follow, curve 1 corresponds to the scheme of real explosion, curve 2 corresponds to instantaneous wave detonation, and curve 3 corresponds to the calculations [9]. In the case of interaction of the products of explosion with the medium the pressure drops to $165.5 \cdot 10^{8} \mathrm{~N} / \mathrm{m}^{2}$ according to the first scheme, and to $49.8 \cdot 10^{8} \mathrm{~N} / \mathrm{m}^{2}$ according to the second scheme. The velocity of particles equals 2662 and $1263 \mathrm{~m} / \mathrm{sec}$, respectively. The scheme of real detonation leads to substantially larger values of the parameters. As the wave propagates, the maximum pressure decreases for different schemes with different intensity. At the distance $\bar{R}=2$ the curves of maximum pressure for the schemes being compared intersect, and the parameters calculated according to the scheme of instantaneous detonation become the larger ones. The difference of maximum pressures calculated according to the two schemes decreases from $70 \%$ on the boundary of the gas chanber to $8 \%$ at the distance $\overline{\mathrm{R}}=10$. In Fig. I we have marked by circles the experimental data [11] corresponding to explosions of a charge of trotyl in water; by triangles we have marked the experimental data [12] for charges of TNT. The results of experiments satisfactorily agree with the calculations according to both schemes, but they correspond more closely to the scheme of real detonation.

In Fig. 2 we have shown the distribution of the velocity of particles $u=u(\bar{R})$ behind the front of a shock wave in water and in the gas chamber for the detonation schemes adopted at time instants when the fronts attain the values 1,$2 ; 2$ and 4 . According to scheme 1 ,

when the front is at the point $\vec{R}=1,2$, the velocity of particles from the front to the contact explosion increases almost linearly; in the gas chamber it falls in the case $\overline{\mathrm{R}}=0.5$ to zero and further maintains this value. When the fronts attain $\vec{R}=2$, the velocity after the jump increases; its maximum is located within the limits of the gas chamber. Then it decreases to zero. When the wave fronts are at the point $\overline{\mathrm{R}}=4$, the difference decreases to a few percent, with the exception of a small region in the gas chamber. The velocities of the wave front according to the schemes being compared are different close to the charge and virtually identical at a distance from the charge.

The distribution of pressure $p=p(\bar{R})$ behind the wave front in water and in the gas chamber is presented in Fig. 3. According to scheme 1, when the wave front reaches $\overline{\mathrm{R}}=1,2$ after the jump a pressure drop takes place; then within the limits of the gas chamber an increase and a new fall to the region of constant pressure take place. Along the products of detonation, toward the center of symmetry, there propagates a rarefication wave whosefront at this instant reaches $R=0.8$. Commencing from this distance, and up to the center of symmetry, the influence of the surrounding medium does not manifest itself. According to scheme 2 , when the front reaches $\bar{R}=1.2$, after the jump there occurs continuous increase of pressure both in the medium and in the gas chamber up to the value corresponding to the initial pressure. A rarefication wave propagates along the products of detonation; its front is located at a distance $\overline{\mathrm{R}} \cong 0.5$. When the fronts reach $\mathrm{R}=2$, the pressure distributions according to the two schemes become close to one another, with the exception of the gas chamber. When the fronts have reached $\bar{R}=4$, the pressure in water and in the products of explosion according to both schemes differs by a few percent.

Values of the pressure and velocity of particles at the wave front and on the contact explosion, when the wave fronts attain distances from 1.2 to 10 , are presented in Table 1. The upper values correspond to the scheme 1, and the lower to 2. From the data of Figs. 1-3 and Table 1 it follows that the wave parameters calculated according to the schemes of real and instantaneous wave detonation differ strongly in the near zone, but for $\overline{\mathrm{R}}=10$ have virtually identical values. The approximation adopted leads to insignificant deviations from results obtained according to more complex equations of state of the products of explosion and waves [9].

We consider the results of calculation of wave parameters in the case of a trotyl charge in water-saturated ground. In Fig. 4 we have presented graphs of maximum pressure varying with the distance in the case of real and instantaneous wave schemes of detonation (curves 1 and 2 , respectively). By a dot we have marked the results of calculations accord-


Fig． 3


Fig． 4

TABLE 1

|  | $\begin{aligned} & \text { Coordi- } \\ & \text { nate } \overline{\mathrm{R}}: \end{aligned}$ |  | Pressure$\begin{aligned} & \mathrm{p} \cdot 10^{-5} \\ & \mathrm{~N} / \mathrm{m}^{2} \end{aligned}$ |  | Velocity of par－ ticles u， $\mathrm{m} / \mathrm{sec}$ |  |  | $\begin{aligned} & \text { Coordi- } \\ & \text { nate } \overline{\mathrm{R}} \text {; } \end{aligned}$ |  | Pressure$\begin{aligned} & \mathrm{p} \cdot 10^{-5} \\ & \mathrm{~N} / \mathrm{m}^{2} \\ & \hline \end{aligned}$ |  | Velocity of par－ ticles $u$,$\mathrm{m} / \mathrm{sec}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 等 | $\begin{aligned} & \text { 荅 } \\ & \underset{\sim}{2} \\ & \hline \end{aligned}$ |  |  |  |  | $\begin{gathered} \text { 苛 } \\ \underset{\sim}{\pi} \end{gathered}$ |  | $\begin{gathered} \stackrel{\rightharpoonup}{E} \\ \underset{\sim}{2} \\ \underset{\sim}{\omega} \end{gathered}$ | $\left\|\begin{array}{ccc} 1 & 1 & x \\ 0 & 0 & 0 \\ 0 & 0 \\ 0 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right\|$ |  | $$ |  | $\begin{aligned} & \text { H } \\ & \text { H } \\ & \text { ה } \end{aligned}$ |
| 3,5 3,5 | 1，2 | 1，08 | $\begin{array}{ll}45 & 931 \\ 40 & 386\end{array}$ | 73 971 <br> 39 602 | 1668 | 1498 1091 | 47,6 47,1 | 6，0 | 2.36 2,39 | 320 303 | 2690 | 321 | 148 164 |
| 8,2 8,7 | 2，0 | 1,36 1,38 | $\begin{array}{ll}10 & 200 \\ 14 & 394\end{array}$ | 19 19 19 008 | 950 1067 | 670 668 | 70,5 69,5 | 8，0 | 2，68 | 155 | 1772 1960 | 255 | 102 112 |
| 25，8 |  | 1，98 | 1221 | 5151 | 472 | 253 | 95，0 | 10，0 | 2，97 | 98 | 1287 | 218 | 75 |
| 26，1 | 4，0 | 1，99 | 1406 | 6002 | 485 | 286 | 93，4 |  | 3，01 | 102 | 1400 | 224 | 83 |
|  |  |  |  |  |  |  | － | 10，0 | － | － | 1184 | 250 | 70 |

ing to the scheme of instantaneous waveless detonation for the same ground and for the same explosive［2］．The difference of the initial values of the maximum pressure is large．With the distance from the source of explosion the difference decreases．

In Table 2 we have presented wave parameters in water－saturated ground．The upper values correspond to the real and the lower values to the instantaneous detonation．With propagation of the wave the values of parameters corresponding to these schemes approach one another more rapidly than in the case of the explosion of pentolyte in water．When the front reaches distances equal to $4-6$ ，they virtually coincide．

The graphs of pressure variation with time $p(t)$ in water－saturated ground at distances equal to $1.5,3$ ，and 5 for both schemes are presented in Fig．5．At the distance $\bar{R}=1.5$ the difference of the curves is large，for $\overline{\mathrm{R}}=3$ it is much less，and for $\overline{\mathrm{R}}=5$ it is prac－ tically absent．

The pressure at the front of the shock wave at the first time instants in the case of pentolyte water is less than in the case of trotyl water－saturated ground．This is connected with the difference in the character of compressibility of these media．In the case of large pressures corresponding to the front of the detonation wave，the influence of trapped air， forming 0.04 of the volume，on the overall compressibility of the ground is small．The com－ pressibility in the first place is determined by compressibility of water and the hard com－ ponent（quartz）．Quartz is less compressible than pure water．In the case of low pressures

TABLE 2

|  | $\begin{aligned} & \text { Coordi- } \\ & \text { nate } \mathrm{R} \end{aligned}$ |  | $\begin{aligned} & \text { Pressure } \\ & \mathrm{p} \cdot 10^{-5}, \\ & \mathrm{~N} / \mathrm{m}^{2} \end{aligned}$ |  | Velocity of particles u， $\mathrm{m} / \mathrm{sec}$ |  | $\begin{aligned} & \text { F } \\ & \text { E } \\ & \text { E } \end{aligned}$ | $\begin{aligned} & \text { Coordinate } \\ & \overline{\mathrm{R}} \end{aligned}$ |  | $\begin{aligned} & \text { Pressure } \\ & \mathrm{p} \cdot 10^{-5} \end{aligned}$ |  | Velocity of particles $u$ ， $\mathrm{m} / \mathrm{sec}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{E}{E}$ |  | $\left\|\begin{array}{ll} 1 & 0 \\ 8 & 0 \\ 0 \\ 40 \\ 40 \end{array}\right\|$ |  | $\begin{gathered} \text { oे } \\ \text { ज } \end{gathered}$ |  | $\begin{aligned} & \text { 炭 } \\ & \substack{\text { an }} \end{aligned}$ |  | $\begin{aligned} & \text { 苞 } \\ & 4 \\ & 4 \\ & \hline \end{aligned}$ |  |  | $\begin{gathered} \text { 共 } \\ \text { 垔 } \\ \text { 筑 } \end{gathered}$ |  |  |
| 3.7 |  | 1，05 | 58230 | 88605 | 1037 | 1074 | 72.1 |  | 2.40 | 270 | 926 | 234 | 49 |
| 4,2 | 1.2 | 1，04 | 38680 | 37060 | 741 | 596 | 71，5 | 6，0 | 2，40 | 321 | 939 | 236 |  |
| 9，6 |  | 1.28 | 10996 | 17490 | 668 | 357 | 122，9 |  | 2，90 | 135 | 420 | 175 | 30 |
| 10，8 | 20 | 1,29 | 13109 | 16607 | 668 | 345 | 122，7 | 8，0 | 2，91 | 125 | 425 | 176 |  |
| 33，3 |  | 1，84 | 1022 | 3469 | 352 | 118 | 209，5 |  | 3，54 | 55 | 214 | 120 | 21 |
| 33,4 | $\therefore$ | 1，84 | 958 | 3661 | 356 | 120 | 209，3 | 10，0 | 3，54 | 56 | 213 | 121 | 21 |



Fig． 5
quartz and water are but little compressible，and the overall compressibility of water－ saturated ground is determined by the compressibility of trapped air；and it turns out to be greater than the compressibility of pure water．

With increase of compressibility of the medium，the intensity of dying away of the wave with distance increases．Earlier such results have been obtained for the same media on the basis of the scheme of instantaneous waveless detonation．They are confirmed by tests ［1］．The radius of the gas chamber in a water－saturated ground increases with time more rapidly than in water．

The analysis of the results of calculation shows that the parameters of explosion waves in solid media at close distances from the place of explosion substantially depends on the scheme of detonation of the explosive adopted．The scheme of instantaneous detonation leads to smaller values of the maximum pressure and velocity of particles than the scheme of real detonation．With distance the values of parameters calculated according to these schemes approach one another．Then，commencing from a certain distance，larger values are assumed by parameters determined according to the scheme of instantaneous detonation．With
further propagation of the wave the difference of parameters again decreases. For the pentolyte and water combination the values of pressure at the front of the wave, calculatedaccording to the two schemes, differ by $5 \%$ for $\overline{\mathrm{R}}=12$, while for the trotyl and water-saturated ground they do so for $\overline{\mathrm{R}}=4$.

Application of the two-term isentropic equation to the explosion products and the Tate equation to water leads to values of parameters of the shock wave which only by a few percent differ from calculations according to more complex nonbarotropic equations of these media [9].

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